CONTROL OF ALTERNATIVE COMPONENTS IN A FUZZY ENVIRONMENT

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CHAPTER 1

INTRODUCTION

1.1. Control in Manufacturing

In any manufacturing situation, the natural variability of a process results in parts manufactured with variable dimensions. This variability of dimension of a part can be expressed in terms of tolerance. It is possible to reduce the tolerance to a minimum by applying proper control over the manufacturing process. But the cost of obtaining a higher degree of control in terms of smaller tolerances mounts rapidly (see Fig. 1.1) [8]. The costs can also be reduced by smart design with manufacturing in mind, the choice of an especially appropriate machine, an efficient quality control system, or quantity production with special machines [8].

In situations where complete control over the process is not possible, or extremely prohibitive in costs, it is a standard practice to have many tolerance groups for a part depending upon the clearance requirements of the mating part. In such cases, it is necessary to manufacture or order the mating components with corresponding tolerances.

Tolerance: There are two extreme permissible sizes for a dimension. The largest possible size for the dimension is known as the upper limit whereas the smallest permissible

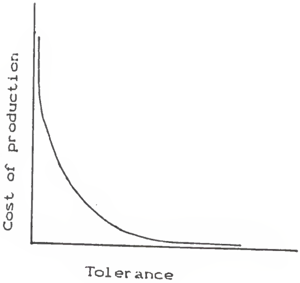


Fig. 1.1. Graph of cost of production vs tolerance [8]

size is called the lower limit. The difference between the upper and lower limit of a dimension is called the tolerance.

Tolerance can be expressed as follows:

Maximum limit = Basic size + upper tolerance.

Minimum limit = Basic size + lower tolerance.

1.2. Alternative Components

Alternative components are the components with the same basic dimension and tolerance but with different upper and lower limits. Main component is one of the alternative components with its dimension closest to the basic dimension and whose incidence is largest among all the alternative components.

Example: Dimensions of a 40 mm. diameter shaft made on a turning machine vary from 38.7 to 40.3 mms. This shaft mates with a bearing whose clearance requirement is 0.2 mms. Now, there are 3 tolerance groups of the same shaft namely, grade 1: min. dia = 39.7 mms., max. dia = 39.9 mms., i.e., 40-0.3 to 40-0.1 mms., grade 2: 40-0.1 to 40+0.1 mms., grade 3: 40+0.1 to 40+0.3 mms. 40 mm. is the basic dimension. Grade 1, grade 2, and grade 3 form the alternative components for the shaft with 40.0 mm diameter. The main component has the dimension 40-0.1 to 40+0.1 mms.

Some common examples of alternative components are: pistons with the same basic diameter and tolerance but with different upper and lower limits, bearings with the same inner diameter and tolerance but with different upper and lower limits.

1.3. Problems With Alternative components

Dealing with many alternative components instead of one component is a very difficult task, especially in planning and control of production. Alternative components result in wrong planned order quantities in Materials Requirement Planning (MRP). If the Bills of Materials (BOM) used for calculating Work-in-Process (WIP) quantities is the same as the one used for MRP, then WIP figures are also inaccurate. This problem is discussed in more detail in chapter 2.

In an automobile industry, it is common to see a large number of components each having several alternative components. But such a large number makes it almost impossible to plan and control the production of these components. In cases where achieving complete control over manufacturing processes is prohibitive in costs or impossible, it is possible to minimize the production of undesirable alternative components and thus maximize the production of main components by the proper choice of machines available. If a component is made on several similar machines, then allocating maximum order quantity to machines which have a higher degree of control would result in a larger number of main components being produced. But given a machine, it is extremely difficult to determine its degree of control over the processes, especially in the absence of past data. In such cases, it is possible to estimate approximately the degree of control (in terms of percentage production of alternative components) based on the decision maker's experience and subjective evaluation. Under such conditions, it is also difficult to quantify goals and constraints precisely. Such an imprecision in the system gives rise to a situation where decisions have to be made in a fuzzy environment. Since the process of optimizing the quantity of desirable components is ongoing and the goal is to be achieved over the length of a number of time periods, it becomes a multi-stage decision-making process. The most common tool used in multi-stage decision-making is Dynamic Programming and it is widely used in our study. we assume a deterministic system under control to start with and solve the problem of alternative components using Dynamic Programming. Later, we fuzzify the goals and constraints to solve the same problem. In the next step, states and controls are also fuzzified to obtain the solution for a further complicated system.

The purpose of this study is to obtain a mathematical model of the system and devise suitable methods to maximize the production quantity of the main component assuming

1. a deterministic system under control
2. a deterministic system under control in a fuzzy

environment.

3. a fuzzy system under control in a fuzzy environment.

1.4. Overview of this report

The next section in this chapter deals with the literature review of various methods available for multi-stage decision-making with varying degrees of fuzziness. Application of these methods to various systems are also discussed.

Chapter 2 discusses the nature of the problem in detail. A mathematical model of the problem is formulated using dynamic programming in a deterministic environment. An example of the problem is also solved using the model.

Chapter 3 presents a basic overview of fuzzy set theory. It also contains all the basic definitions, operations, and techniques in fuzzy set theory which are used in this study.

In chapter 4, the mechanism of multi-stage decision-making is presented. An example of the problem is solved using Bellman-Zadeh model assuming the crisp system under control in a fuzzy environment.

In chapter 5, the problem is considered to be a fuzzy system under control in a fuzzy environment. Two methods of fuzzy dynamic programming with fuzzy control and state variables are discussed with their relative merits and de-merits. An example of the problem is also solved using a modified approach.

Chapter 6 summarizes the study and the results. A discussion on the results follows.

1.5. Literature Review

This section surveys the current state of literature on multi-stage decision-making in a fuzzy environment. Though the problem of alternative components is well known as a planner's nightmare and is almost unavoidable in a precision manufacturing system, there is no evidence of this problem ever being studied at all. There is no literature available on the effect of alternative components on MRP and WIP and their inclusion in the BOM.

Since its introduction in 1973 by Joseph Orlicky [21], Materials Requirement Planning (MRP) has replaced all the previously existing planning methods. Now, it is a standard and most effective planning tool available today. This prompts us to study the effect of alternative components on MRP.

Since the publication of the innovative paper "Fuzzy sets" by L.A. Zadeh in 1965, more than 4000 papers have been published by the researchers in this area. Bellman and Zadeh [3] used this new concept of fuzzy sets to introduce the process of decision-making in a fuzzy environment. They contend the practice of solving problems involving imprecision by equating imprecision to randomness. To quote a few words from them "Much of the decision-making in the real world takes place in an environment in which goals, the constraints, and the consequences of possible actions are not known precisely. To deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory and, more particularly, the tools provided by the decision theory, control theory and information theory. In doing so, we are tacitly accepting the premise that imprecision, whatever its nature, can be equated with randomness. This, in our view, is a questionable assumption".

"Specifically, our contention is that there is a need for differentiation between randomness and fuzziness, with the latter being a major source of imprecision in many decision processes...". Bellman and Zadeh formulated a fuzzy decision model in which a fuzzy decision is a confluence of fuzzy goals and fuzzy constraints. They also developed a multi-stage decision-making model in a fuzzy environment using dynamic programming. This model is used as a basic tool in our study. Fuzzy dynamic programming is a natural extension of dynamic programming in conjunction with fuzzy sets.

Bellman and Zadeh's model provides a basic framework for multi-stage decision-making in a fuzzy environment where the system under control is either deterministic or stochastic with implicitly or explicitly defined termination time. A further study on stochastic system under control in a fuzzy environment with a fuzzy termination time was done by Kacprzyk in [8].

A dynamic programming model in a fuzzy environment where the system under control is fuzzy, is presented by Baldwin and Pilsworth in [1]. They use a fuzzy state mapping to denote the fuzziness of control and state variables. In order to overcome the high dimensional state, they use a method of approximation where fuzzy reference sets for both control state are used by making use of conditional fuzzy statements of the IF......THEN form. Kacprzyk presents a much simpler method [10] using branch-and-bound algorithm with state and control as fuzzy.

Applications of fuzzy dynamic programming are novel and diverse. Esogbue and Bellman [7] studied the application of fuzzy dynamic programming to a wide variety of problems like the budgeting problem in cancer research [4], R & D management [5], health care delivery problem involving anesthesia administration problem [7], ill defined energy systems [7], and water resource system and cluster analysis [7]. Kacprzyk and Straszak [11] apply fuzzy dynamic programming in formulating the problem of optimal control of regional development trajectories. Kacprzyk and Staniewsky [12] made use of fuzzy numbers in applying dynamic programming to long-term inventory policy-making in a fuzzy environment to obtain optimal inventory policies. Samoylenko [22] used the fuzzy dynamic programming for optimization problems associated with computer network design and routing. Moore et al. [18] used the fuzzy dynamic programming in automatic speech recognition. The field is nevertheless still embryonic.

CHAPTER 2

ALTERNATIVE COMPONENTS IN PLANNING AND CONTROL - A MODEL

2.1. INTRODUCTION

As we have seen in section 1.1, inability to achieve complete control over manufacturing processes results in alternative components. Alternative components pose many problems in an industrial environment. Some of them are: representation in the Bills of Materials, wrong requirement quantities in MRP, and wrong estimation of WIP. The next section in this chapter discusses all the above problems. A problem involving alternative components encountered in an automobile manufacturing industry is presented. A mathematical model of this problem is formulated using dynamic programming approach. An example of this problem is also solved making use of the model.

2.2 Representation of alternative components in BOM

Since alternative components are also inventory items [21] and form many options for the same part, it is essential to represent them appropriately in BOM.

Consider the example given in section 1.1. Suppose this shaft has to be fitted with a bearing to form a sub-assembly and this sub-assembly has to be used in an assembly with a quantity-per [21] of 1. Then, there would be 3 options for the bearing since bearing has 3 alternative shafts.

One way of representing alternative components in BOM is to define 3 unique assemblies with each assembly having each of these three alternative shafts for the bearing. If we follow the same logic, there would be 3 options for the end-product. If there is a large number of parts each of which having several alternative components, or if each of the alternative sub-assemblies have, in turn, many parts having many alternative components, then, the number of options for the end-product would be enormously high.

Another way is to represent the main component with the usual quantity-per and the rest of the alternative components with quantity-per as zero. This will ensure that all the inventory items are represented in BOM. But while calculating the requirements via MRP logic, there would be requirements for only the main component and the rest of the alternative components would have a requirement of zero.

The standard practice is to represent all the alternative components under the same assembly with the quantity-per being split among the alternative components in proportion with their past usage (see Fig. 2.1 and 2.2). With this arrangement, it is possible to avoid numerous end-products. This would enable to calculate the requirements of all the alternative components for the assembly. This practice is, however, not fool-proof and results in wrong MRP and WIP figures when the degree of control over manufacturing processes for the part is not predictable.

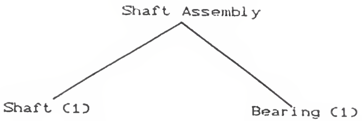


Fig. 2.1

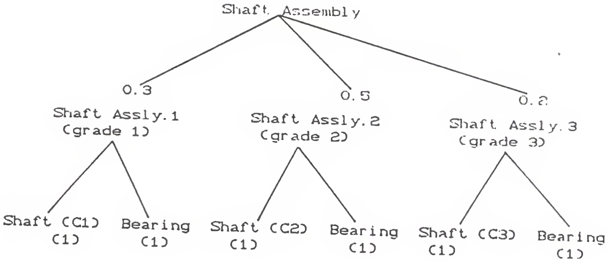


Fig. 2.2. Representation of Alternative Components in BOM

2.3 Wrong Requirements in MRP

Let the requirement of the assembly be 1000 units. Let the inventory levels of C1, C2, and C3 be 400, 100, and 400 numbers respectively. Requirements of alternative components are calculated as below:

Gross Requirement = Net Requirement of the sub-assembly * Quantity-per of the component

Net Requirement = Gross Requirement - Stock-on-hand

C1: Gross Requirement = 1000 * 0.3 = 300

Net Requirement = 300 - 400 = -100

= 0, since we already have 100 in excess.

C2: Gross Requirement = 1000 * 0.5 = 500

Net Requirement = 500 - 100 = 400

To be ordered = 400

C3: Gross Requirement = 1000 * 0.2 = 200

Net Requirement = 200 - 400 = -200

= 0 since we already have 200 in excess.

In overall, to make 1000 assemblies, we needed 1000 numbers of shafts. We already have 900 on hand. So, we needed to order only 100 numbers. But we ended up ordering 400 !

2.4. Wrong WIP figures

If the consumption of C1, C2, and C3 are different than the assumed quantity-per, then WIP quantities of alternative components calculated using quantity-per in BOM are inaccurate.

Let the number of sub-assemblies made be 1000. Let the actual consumption of C1, C2, and C3 be 400, 300, and 300 respectively. If we have already issued 500 numbers of each component to the assembly shop, then

i. Calculation using BOM

C1: WIP = 500 - 1000 * 0.3 = 200

C2: WIP = 500 - 1000 * 0.5 = 0

C3: WIP = 500 - 1000 * 0.2 = 300

ii. Actual computations

C1: WIP = 500 - 400 = 100

C2: WIP = 500 - 300 = 200

C3: WIP = 500 - 300 = 200

In actuality, WIP's of C1, C2, and C3 are 100, 200, and 200 numbers respectively. But according to the calculations made using quantity-per in BOM, these figures are 200, 0, and 200 respectively. So, the WIP figures are wrong!

2.5. Dynamic Programming Approach

The problems due to alternative components discussed in the previous section are due to the variability of process on machines. The variability could be due to some intrinsic characteristics of the machine, or due to the skills of the operator working on the machines. In cases where the parts are made on many similar machines for the purpose of mass production, it would be appropriate to assign the order quantities to the machines in proportion to their degrees of control so that the overall degree of control is maximized. This would obviously result in the maximum production of the most desirable component, thus minimizing the effect of alternative components on MRP and WIP. In cases where the part is made on only one machine in many shift with many operators working on them, assigning the order quantities to the operators in proportion with their degree of control over the machines would result in maximizing the quantity of the most desirable component. The order quantities assigned to different machines or different operators constitute the alternative plans. Since this process of maximizing is ongoing and takes place over the length of finite number of periods, it is a multi-stage decision process. The alternative plans would be the control variables and the quantities produced of alternative components during each period would be the state variables. This is an ideal situation for applying dynamic programming.

2.6. Dynamic Programming Model - In a Crisp Environment

Consider the example shown in section 1.1 and the standard BOM representation of alternative components as in section 2.2. Let us assume that the shafts are manufactured on 2 similar machines for the purpose of mass production, each producing its own mix of alternative components.

For the sake of simplicity, let us assume that the capacity of both the machines be B_l per period, where l is the number of machines = 1,2, and let the minimum number that must be produced on each machine be b_l

Assumptions:

1. Total demand never exceeds total capacity.
2. Demand not met during the previous period is carried over to the next period unless explicitly stated.
3. Initial inventory is zero.
4. Production quantity is always in multiples of 100.

Let Z_j = Objective function value for the period j.

w_i = weight given to the alternative component i in the objective function.

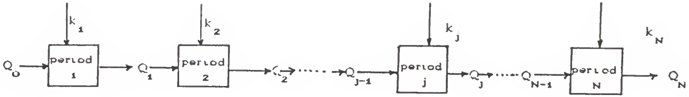
x_j = inventory at the end of period j.

Q_{jkl} = Total quantity produced during the period j for the alternative k on the machine l.

s_{il} = production percentage of alternative component i on the machine l.

q_{ijk} = quantity produced during period j of the alternative component i for the alternative k

d_{ij} = consumption of alternative component i for the period j.



Where k_j is the number of alternatives for the j^{th} period

Q_j is the quantity to be produced during j^{th} period

Fig. 2.3 A multi-stage decision process

D_j = Total demand for the part for the period j.

P_j = Planned requirement for the period j.

O_j = net quantity to be produced during the period j.

I_{ijk} = Inventory of the alternative component i at the end of the period j for the alternative plan k.

Since our objective is to maximize the quantity of the most desirable component based on the weights of desirability, we have the objective function

$$\text{Max } Z_j = \sum_{i=1}^{n} w_i q_{ijk} \qquad \ldots\ldots(2.1)$$

Now, $f(q_{i(j-1)k})$ = max. value of Z from stages j,j+1,...,n-1,n with $q_{i(j-1)k}$ as the quantity produced of alternative component i for the periods beginning from j, j+1,..., n-1, n.

Since the quantity produced of each alternative component depends upon the percentage production factor for each machine,

$$q_{ijk} = s_{il} * Q_{jkl} \qquad \ldots\ldots\ldots\ldots(2.2)$$

Inventory remaining for the alternative component i at the end of the period j for proposal k = This period's production + Previous period's inventory - this period's consumption , i.e.,

$$I_{ijk} = q_{ijk} + I_{i(j-1)k} - d_{ij} \qquad \ldots\ldots\ldots(2.3)$$

Total inventory for the part (inclusive of all alternative components) = $x_j = \sum_{i=1}^{n} I_{ijk}$ ....(2.4)

$$f(q_{i(j-1)k}) = \max_{k_j} \max_{k_{j+1}} \ldots \max_{k_n} [Z_j + Z_{j+1} + \ldots + Z_n]. \qquad \ldots(2.5)$$

Since the process flow is forward, stage i is calculated first before calculating stages j+1, j+2, ..., n. Thus, the results of stage j will influence the results of stage j+1, J+2, ..., n, but not vice-versa.

$$\therefore f_j(q_{i(j+1)k}) = \max \left[Z_j + \max_{k_{j+1}} \ldots \max_{k_n} \left\{ Z_{j+1} + \ldots + Z_n \right\} \right]$$

$$\text{but } \max_{k_{j+1}} \ldots \max_{k_n} \left\{ Z_{j+1} + \ldots + Z_n \right\} = f_{j+1}(q_{ijk})$$

$$\therefore f_j(q_{i(j+1)k}) = \max_{k_j} \left[Z_j + f_{j+1}(q_{ijk}) \right] \quad \ldots\ldots (2.6)$$

Now, net quantity to be produced during the period j = Quantity planned for the period j - inventory at the end of the period j-1, i.e.,

$$O_j = P_j - x_{j-1} \quad \ldots\ldots\ldots (2.7)$$

$$Q_{jk1} = O_j - k(100) - b_1 \quad \ldots\ldots\ldots (2.8)$$

$$Q_{jk2} = O_j - Q_{jk1} \quad \ldots\ldots\ldots\ldots (2.9)$$

The equations 2.8 and 2.9 can be used for obtaining various production alternatives for each period which is based on the inventory remaining after the previous period (x_j) and the planned requirement (P_j) for the current period.

2.7. Solution Using Dynamic Programming Model

We are going to use the dynamic programming model derived in the previous section to solve a problem. For the sake of simplicity, the computations will be made over a length of 2 periods.

Assume a production plan P_1 = 1000 units for the first period and $P_2 = 900$ for the second period. Let the maximum production capacities of the machines be $B_1 = B_2 = 600$ units. Let the minimum batch quantity for each machine $b_1 = b_2 = 400$ units. Let the demand for period 1, $D_1 = 1000$ units with the quantity split up among C1, C2, and C3 as 15%, 65%, and 20% respectively. $D_2 = 900$ with the same split up. Let the percentage production factors (s_{il}) for each component on each machine be as in table 2.2.

Now, the first step is to calculate various production alternatives. This is done by using 2.8 and 2.9.

$P_1 = 1000,\ j = 1.$

Plan 1: $Q_{111} = 1000 - 0(100) - 400 = 600$

$Q_{112} = 1000 - Q_{jk1} = 1000 - 600 = 400.$

Plan 2: $Q_{121} = 1000 - 1(100) - 400 = 500$

$Q_{122} = 500$

Plan 3: $Q_{131} = 400$

$Q_{132} = 600$

This is shown in table 2.3.

Now, let us assume weights for the alternative components. Since C2 is the most desirable one, we give a weight of 1. Let the weights for C1 and C3 be -0.8 and -0.6 respectively. The negative sign indicates the undisirability. Then the objective function becomes

$$\max Z_j = -0.8q_{1jk} + q_{2jk} - 0.6q_{3jk}$$

Table 2.4, 2.5, and 2.6 show the detailed calculations.

Refering to table 2.6, we get

max. value of $Z = 807$, $k^* = (k_3^0, k_4^1)$, i.e.

Period	Qty. to be assigned to	
	machine 1	machine 2
1	400	300
2	600	600

Now, suppose we define some constraints and goals for the deterministic system as follows:

Goal: Maximize Z > 750

Constraints:

1. Shortage of each alternative components must be less than 10% of demand.

2. Quantity of C2 manufactured must exceed 65%.

3. Overall utilization of each machine must be > 85%.

The alternative (k_3^0, k_4^1) satisfies the goal and the constraints 2 & 3 but the shortage of C3 for both the periods is much higher than 10%. Looking through the alternatives in table 2.4 and 2.6, we do not find any alternatives which satisfy all the costraints and the goal. The constraints seem to be conflicting each other. This means that the decision maker does not have any choices that strictly meet all his requirements. So, this situation does not provide any rational basis to make decisions. The best way to get out of this deadlock is to choose the alternative which satisfies the goals and constraints as close as possible. The term "as close as possible" is highly subjective and indicates that there is fuzziness involved in the environment in which one has to make a decision. Next chapter deals with decision making in a fuzzy environment.

Table 2.2. Percentage Production figures of alternative components on two machines.

Comp Machine	C1	C2	C3
1	15	60	25
2	20	70	10

Table 2.3. Production plan alternatives for period 1

Machines	Alternatives		
	1	2	3
1	600	500	400
2	400	500	600

Table 2.4. Dynamic Programming Table for Stage 0.

Altern-atives (k)	Quantity produced this period $q_{ijk} = s_{il} \times Q_{jkl}$			Z $Z = \sum_{i=1}^{n} w_j q_{ijk}$	Inventory remaining after consumption $q_{ijk} - d_{ij}$			Total net requirement for for next period
	C1	C2	C3		C1	C2	C3	
1	90+80 =170	360+280 = 640	150+40 = 190	390	20	-10	-10	900
2	175	650	175	396	25	0	-25	900
3	180	660	160	420	30	10	-40	900

Table 2.5. Production plan alternatives for period 2

Machines	Alternatives 1	 2	 3	 4
1	600	500	400	300
2	300	400	500	600

Table 2.6. Dynamic Programming Table for Stage 1

Plan for Previous Period	Plan for this period	Quantity produced this period			Z	Inventory remaining after consumption			Z for previous period	Overall Z
		$q_{ijk} = s_{il} \times Q_{jkl}$			$Z = \sum_{i=1}^{n} w_j q_{ijk}$	$q_{ijk} + I_{i(j-1)k} - d_{ij}$				
k^0	k^1	C1	C2	C3		C1	C2	C3	Z^0	$Z^0 + Z^1$
1	1	90+60 = 150	360+210 = 570	150+30 = 180	342	150+20 -135 = 35	570-10 -585 = -25	80-10 -180 = -10	390	732
	2	155	580	165	357	40	-15	-25	390	747
	3	160	590	150	372	45	-5	-40	390	762
	4	165	600	135	387	50	5	-55	390	777
2	1	150	570	180	342	40	-15	-25	396	738
	2	155	580	165	357	45	-5	-40	396	753
	3	160	590	150	372	50	5	-55	396	768
	4	165	600	135	387	55	15	-70	396	783
3	1	150	570	180	342	45	-5	-40	420	762
	2	155	580	165	357	50	5	-55	420	777
	3	160	590	150	372	55	15	-70	420	792
	4	165	600	135	387	60	25	-85	420	807

CHAPTER 3

FUNDAMENTALS OF FUZZY SET THEORY

3.1. Introduction

So far we considered the situation in which the decision environment and the parameters were known. However this need not be always the case and some sort of uncertainty may exist. Some of the distinguished cases under which a decision is made are:

-Certainty
-Risk
-Uncertainty
-Imprecision (fuzziness)
-Ignorance

Risk and uncertainty can be handled by the application of probability theory. In case of imprecision, the state can not be well defined because of lack of complete knowledge and the vagueness of the state. For example, consider the decision situation where the decision-maker wants to keep the inventory levels as low as possible. The term " as low as possible" is a vague term and is not precise. In some cases, it is impossible to assign a specific value to this term because of the impreciseness or fuzziness involved in describing the state inventory levels. Some other examples of fuzzy terms are, a set of "tall" persons, a set of "big" cars etc. In such cases the classical probability theory can not be applied because it can handle only random situations rather than vague situations. Moreover, in the absence of past data, probability theory can not be applied as there would be no basis on which probabilities can be estimated. Many of the situations encountered in practice come under this category. The major breakthrough came when L.A. Zadeh introduced fuzzy set theory. It enabled the researchers to model mathematically the situations involving fuzziness. Basically its idea is as follows: The conventional set may be characterized by its characteristic function which takes on values only 0 and 1, such that 0 means that a particular element does not belong to the set, and 1 means that it belongs. The elements may therefore either belong or not to the set, i.e. the set boundaries are crisp. In a fuzzy set, the characteristic function is replaced by the membership function which takes on its values in the whole interval [0,1]. The values of membership function express the degree to which elements belong to the fuzzy set. The boundary of a fuzzy set is therefore not clear cut, and this is virtually what we need to characterize fuzziness. The theory of fuzzy sets provides elegent, adequate and simple enough tools to accommodate fuzziness in mathematical modeling.

This chapter discusses the fundamentals of fuzzy set theory and the basic concepts of the operators and tools that are used throughout this study.

3.2. Some Basic Concepts And Definitions

3.2.1. Fuzziness

Fuzziness, in the mathematical sense, is a description of vagueness. However, fuzziness should not be mistaken for ambiguity, uncertainty, or imprecision. An ambiguity deals with several competing descriptions of a set of activities or observations. Uncertainty is the use of random variables to represent possible values of activities or observations in a well defined set. Imprecision implies the existence of a sharp tolerance interval for the description of a set of activities or observations. Fuzziness occurs when the physical constraints defining the tolerance are themselves elastic or ill-defined.

3.2.2. Fuzzy Sets

A fuzzy set is a class of objects (or activities or observations) paired with their respective degrees of membership within the set.

The degrees of membership usually have the range [0,1] with 1 denoting total or complete membership.

Mathematically, if X is a collection of objects denoted by x, then a fuzzy set $A = \{(x, \mu_A(x))\ x \in X\}$

$\mu_A(x)$ is called the membership function or degree of membership of x in A.

Fuzzy sets are represented in two ways. One way is to represent them in the form of $A = \{x, \mu_A(x)\}$. The other way is to represent them as $A = \{\mu_A(x_1)/x_1 + \mu_A(x_2)/x_2\ + \ldots\}$

Example:

$A = \{(1, 0.1), (2, 0.2), (3, 0.6), (4, 1)\}$

$A = \{0.1/1 + 0.2/2 + 0.6/3 + 1/4\}$

Both representations are used in this study.

3.2.3. Basic Properties And Operators

1. Properties

a. Containment

A fuzzy set $A \underset{\sim}{\subset} X$ is said to be contained in a fuzzy set $B \underset{\sim}{\subset} X$, written $A \leq B$, if and only if

$$\mu_A(x) < \mu_B(x), \qquad \forall\, x \in X$$

b. Normality

A fuzzy set $A \underset{\sim}{\subset} X$ is said to be normal if and only if

$$\max_{x \in X} \mu_A(x) = 1$$

c. Support

The support of a fuzzy set $A \underset{\sim}{\subset} X$, written as Sup A is defined as $\text{Sup } A = \{x \in X: \mu_A(x) > 0\}$

d. Convexity

A fuzzy set A is convex if

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geqslant \min\,[\mu_A(x_1), \mu_A(x_2)]$$

$\forall\, x_1, x_2 \in X$ and $\lambda \in [0,1]$ where min denotes minimum

e. α-Level Set

The (crisp) set of elements which belong to the fuzzy set A at least to the degree α is called the α-level set.

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$$

$A'_\alpha = \{x \in X \mid \mu_A(x) > \alpha\}$ is called "strong" α-level set, or strong α-cut.

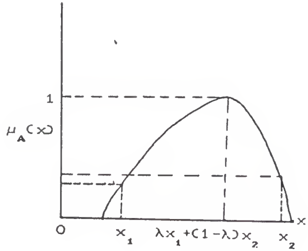


Fig. 3.1. A convex fuzzy set

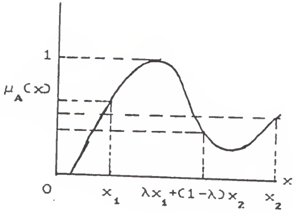


Fig. 3.2. A non-convex fuzzy set

2. Operations

As in the conventional set theory, the basic operations in fuzzy set theory are the complement, union and intersection.

a. Complement

The complement of a fuzzy set $A \underset{\sim}{\subset} X$, written $\neg A$, is defined as $\mu_{\neg A}(x) = 1 - \mu_A(x)$, $\quad \forall\, x \in X$

see Fig. 3.2.

b. Union

The union of two fuzzy sets, $A, B \underset{\sim}{\subset} X$, written $A + B$, is defined as

$$\mu_{A+B}(x) = \mu_A(x) \vee \mu_B(x), \qquad \forall\, x \in X$$

where "$\vee$" is the maximum operator

Example:

If $X = \{1,2,3,4\}$, and $A = 0.2/1 + 0.5/2 + 0.8/3 + 1/4$ and

$B = 1/1 + 0.8/2 + 0.5/3 + 0.2/4$, then

$A + B = 1/1 + 0.8/2 + 0.8/3 + 1/4$

see Fig. 3.3.

c. Intersection

The intersection of two fuzzy sets A, $B \underset{\sim}{\subset} x$, written $A \cap B$, is defined as

$$\mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x), \qquad \forall\, x \in X$$

where "$\wedge$" is the minimum operator.

Example:

If $X = \{1,2,3,4\}$ and $A = 0.2/1 + 0.5/2 + 0.8/3 + 1/4$ and $B = 1/1 + 0.8/2 + 0.5/3 + 0.2/4$, then

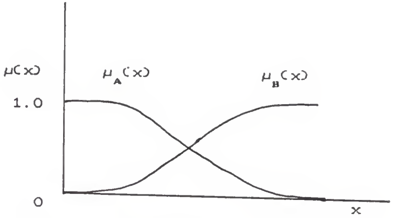


Fig. 3.3. Complement

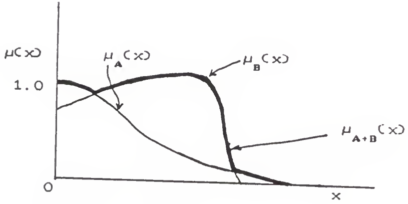


Fig. 3.4. Union

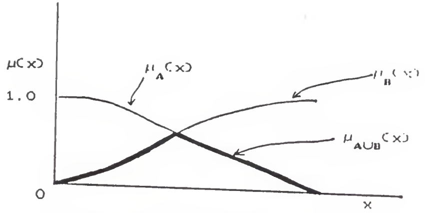


Fig. 3.5. Intersection

$A \cap B = 0.2/1 + 0.5/2 + 0.5/3 + 0.2/4$

see Fig. 3.4.

3.2.4. Fuzzy Numbers

A fuzzy number N is a convex normalized set of the real line $\mathbb{R}$ such that

1. There exists exactly one $x_m \in \mathbb{R}$ whose membership function $\mu_N(x_m) = 1$.
2. $\mu_N(x)$ is piecewise continuous where x_m is the mean of the fuzzy number.

Example:

The following fuzzy set is a fuzzy number A = {numbers close to 5} with the membership function

$$\mu_A(x) = \begin{cases} (-(x-5)^2)/4 + 1, & 3 < x < 5 \\ x/3 + 8/3 & 5 < x < 8 \\ 0 & \text{elsewhere} \end{cases}$$

3.2.5. Triangular Fuzzy Numbers

The triangular fuzzy number is derived from the shape of the membership function graph. The graph is composed of 2 linear functions joined at the mean value x_m. In general, a triangular fuzzy number has the membership function of the form

$$\mu_A(x) = \begin{cases} (x-a_1)/(a_2-a_1) & a_1 < x < a_2 \\ (a_3-x)/(a_3-a_2) & a_2 < x < a_3 \\ 0 & \text{elsewhere} \end{cases}$$

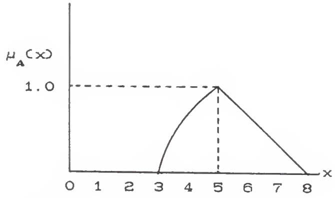


Fig. 3.6. Numbers close to 5

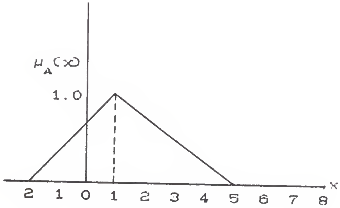


Fig. 3.7. Graph of the triangular fuzzy number (numbers close to 1)

Example:

A = { numbers close to 1} with a membership function of

$$\mu_A(x) = \begin{cases} x/3 + 2/3, & -2 \leqslant x \leqslant 1 \\ -x/4 + 5/4, & 1 < x \leqslant 5 \\ 0 & \text{elsewhere} \end{cases}$$

3.2.6. Fuzzy Relations

The relation is one of the key concepts in mathematics. The same is true for fuzzy relation in fuzzy mathematics.

A fuzzy relation R between two non-fuzzy sets X and Y is a fuzzy set in the cartesian product X × Y, i.e. $R \underset{\sim}{\subseteq} X \times Y$. It is defined as

$$R = \{(x,y),\ \mu_R(x,y)/(x,y)\} \qquad \forall\ (x,y) \subseteq X \times Y$$

Example:

Let X = {Horse, Donkey} and Y = {Mule, Cow}.

The fuzzy relation R labeled "similarity" may be as follows

R = "similarity" = 0.8/(Horse, mule) + 0.4/(Horse, cow) + 0.9/(Donkey, Mule) + 0.2/(Donkey, Cow)

The particular grades of membership give the degree to which the respective animals are similar, e.g., a horse and a mule are similar to the degree 0.8, while a donkey and a cow are similar to the degree 0.2 only. It can be represented in the matrix form

$$R = \text{"similarity"} = \begin{array}{c|cc} & \text{mule} & \text{Cow} \\ \hline \text{horse} & 0.8 & 0.4 \\ \text{donkey} & 0.9 & 0.2 \end{array}$$

The fuzzy relation is evidently a fuzzy set and all the fuzzy set properties, operations, etc. are applicable. One of the most important property is the max-min composition.

Consider relations $R \underset{\sim}{\subset} X \times Y$ and $S \underset{\sim}{\subset} Y \times Z$. It is expressed as $R \circ S \underset{\sim}{\subset} X \times Z$, such that

$$\mu_{R \circ S}(x,z) = \max_{y \in Y} (\mu_R(x,y) \wedge \mu_S(y,z)) \qquad \forall\, x \in X,\ z \in Z$$

Example:

If $X = \{1,2\}$, $Y = \{1,2,3\}$, and $Z = \{1,2,3,4\}$, then the composition of two fuzzy relations R and S is

$$\begin{array}{c} R \\ y \\ x \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.3 & 0.8 & 1 \\ 0.9 & 0.7 & 0.4 \end{bmatrix} \end{array} \end{array} \circ \begin{array}{c} S \\ z \\ y \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.7 & 0.6 & 0.4 & 0.1 \\ 0.4 & 1 & 0.7 & 0.2 \\ 0.5 & 0.9 & 0.6 & 0.8 \end{bmatrix} \end{array} \end{array}$$

$$\begin{array}{c} R \circ S \\ z \end{array}$$

$$= \quad x \begin{array}{cc} & \begin{array}{cccc} 1 & 3 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.5 & 0.9 & 0.7 & 0.8 \\ 0.7 & 0.7 & 0.7 & 0.4 \end{bmatrix} \end{array}$$

An example calculation is as below.

$$\begin{aligned} \mu_{R \circ S}(2,3) &= \max_{y \in \{1,2,3\}} (\mu_R(2,y) \wedge \mu_R(y,3)) \\ &= (\mu_R(2,1) \wedge \mu_S(1,2)) \vee (\mu_R(2,2) \wedge \mu_S(2,3) \vee \\ &\quad\ (\mu_R(2,3) \wedge \mu_S(3,2)) \\ &= (0.9 \wedge 0.4) \vee (0.7 \wedge 0.7) \vee (0.4 \wedge 0.8) \\ &= 0.4 \vee 0.7 \vee 0.4 \\ &= 0.7 \end{aligned}$$

3.2.7. The Extension Principle

The extension principle is one of the most important and powerful tools in the theory of fuzzy set. It makes it possible to extend some known models or algorithms involving non-fuzzy variables to the case of fuzzy variables.

Let $A_1 \underset{\sim}{\subseteq} X_1, \ldots, A_n \underset{\sim}{\subseteq} X_n$ be some fuzzy sets and f: $X_1 x \ldots x X_n \rightarrow Y$ be some fuzzy function. Then, due to the extension principle, the fuzzy set $B \underset{\sim}{\subseteq} Y$ induced by fuzzy sets $A_1, \ldots, A_n$ through f is

$$\mu_B(y) = \max \bigwedge_{i=1}^{n} \mu_{A_i}(x_i)$$

$$(x_1, \ldots, x_n) \in X_1 x, \ldots, X_n \; : \; y = f(x_1, \ldots, x_n)$$

Example:

Let $x_1 = \{1,2,3\}$, $x_2 = \{1,2,3,4\}$, f represents the addition, i.e. $y = x_1 + x_2$ and

$A_1 = \{0.1/1 + 0.6/2 + 1/3\}$ and

$A_2 = \{0.6/1 + 1/2 + 0.5/3 + 0.1/4\}$, then

$B = A_1 + A_2 = 0.1/2 + 0.6/3 + 0.6/4 + 1/5 + 0.1/7$

This principle could be extended to other operators like min and max.

3.2.8. Fuzzy Sets of Type 2

A fuzzy set of type 2 is defined by a fuzzy membership function, the grade of which is also a fuzzy set in the unit interval [0,1] rather than a point in [0,1]. In other words, a type 2 fuzzy set whose membership values are type 1 fuzzy sets on [0,1].

The fuzzy membership can be represented as

$\mu_A : X \rightarrow [0,1]^J$ where $\mu_A(x)$ is the fuzzy grade being a fuzzy set in [0,1] (or in the subset J of [0,1]).

3.2.9. Operations on Type 2 Fuzzy Sets

The extension principle can be used to define the basic operations for the type 2 fuzzy sets [18].

Let

$$A(x) = \{(x,\ \mu_A(x))\},\quad B(x) = \{(x,\ \mu_B(x))\}$$

where

$$\mu_A(x) = \{(u_i,\ \mu_{u_i}(x)) \mid x \in X;\ \ u_i, \mu_{u_i}(x) \in [0,1]\}$$

$$\mu_B(x) = \{(v_j,\ \mu_{v_j}(x)) \mid x \in X;\ v_j,\ \mu_{v_j}(x) \in [0,1]\}$$

The u_i and v_j are degrees of membership of type 1 fuzzy sets and $\mu_{u_i}(x)$ and $\mu_{v_j}(x)$ are their membership grades respectively.

1. **Union**

$$\mu_{A \cup B}(x) = \mu_A(x) \cup \mu_B(x)$$

$$= \{w, \mu_{A \cup B}(w) \mid w = \max\ \{u_i,\ v_j\},\ u_i,\ v_j \in [0,1]\}$$

where

$$\mu_{A \cup B}(w) = \sup_{v=\max(u_i,v_j)} \min\ \{\mu_{u_i}(x),\ \mu_{v_j}(x)\}$$

2. **Intersection**

$$\mu_{A \cap B}(x) = \mu_A(x) \cap \mu_B(x)$$

$$= \{(w,\ \mu_{A \cap B}(w) \mid w = \min\ (u_i,\ v_j),\ u_i,\ v_j \in [0,1]\}$$

where

$$\mu_{A\cap B}(w) = \sup_{v=\min(u_i,v_j)} \min\left(\mu_{u_i}(x),\ \mu_{v_j}(x)\right)$$

Type 2 fuzzy numbers follow commutative and associative laws and follow De Morgan laws. However, they do not satisfy distributive, absorption, and complementary laws [18].

CHAPTER 4

DYNAMIC PROGRAMMING APPROACH - IN A FUZZY ENVIRONMENT

4.1. Introduction

In the previous chapter, a dynamic programming model was formulated and a solution to the problem of alternative components was obtained in a crisp (deterministic) environment. Most often the real life situations are not deterministic. Most of the quantities can only be expressed in terms of ranges and the limits of these ranges are not fixed but are vaguely expressed. In such situations, it would be most appropriate to solve the problems using fuzzy set theory. This chapter deals with the process of decision-making in a fuzzy environment. Bellman-Zadeh's model of dynamic programming in a fuzzy environment is used as a basis for solving the problem. A branch-and-bound algorithm proposed by Kacprzyk is also dealt with. A fuzzy solution is obtained for the problem of alternative components using the branch-and-bound approach.

4.2. Decision-Making in a Fuzzy Environment

A general conceptual framework for decision-making in a fuzzy environment was formulated by Bellman and Zadeh [3] in 1970. This model is extensively used in many fields due to its simplistic approach.

4.2.1. Fuzzy Goal, Fuzzy Constraint, and Fuzzy Decision

A fuzzy environment consists of fuzzy goals, fuzzy constraints, and a fuzzy decision.

To define the above elements, it is necessary to introduce some set of possible options (or choices, alternatives, etc.) denoted by $X = \{x\}$, where x is a generic element of X. The elements of X may be values of expenditures, alternative courses of action, etc. X contains all possible, or all relevent options.

Fuzzy goal: Fuzzy goal G is defined as a convex fuzzy set in the set of options X, i.e., $G \underset{\sim}{\subset} X$ characterized by the membership function $\mu_G(x)$.

For example, a fuzzy goal can be expressed in words as "x should be substantially larger than 10" whose membership function is subjectively given by

$$\begin{aligned} \mu_G(x) &= 0, \quad x < 10, \\ &= (1 + (x - 10)^{-2})^{-1}, \quad x \geqslant 10 \qquad (4.1) \end{aligned}$$

Fuzzy constraint: Fuzzy constraint C is defined as a convex fuzzy set in the set of options X, i.e., $C \underset{\sim}{\subset} X$, characterized by membership function $\mu_C(x)$.

For example, a fuzzy constraint could be "x should be in the vicinity of 15" which can be represented by a membership function

$$\mu_C(x) = (1 + (x - 5)^4)^{-1} \qquad (4.2)$$

Now, the general statement of the decision-making problem in a fuzzy environment is "Attain G and satisfy C". Here, "and" corresponds to the intersection of fuzzy sets.

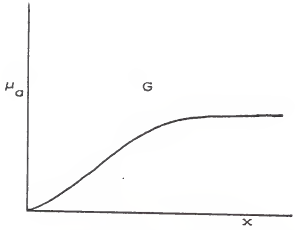


Fig. 4.1. Fuzzy goal: "x should be substantially larger than 10".

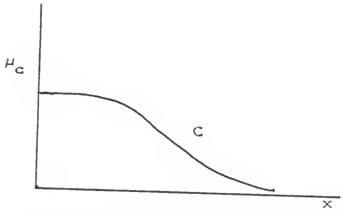


Fig. 4.2. Fuzzy Constraint: "x should be in the vicinity of 15".

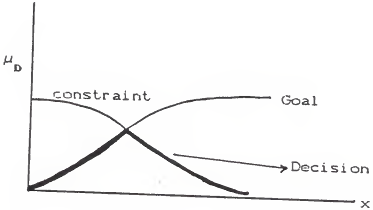


Fig. 4.3. Fuzzy decision

Fuzzy decision: Fuzzy decision can be defined as the set resulting from the intersection of G and C. In symbols,

$$D = G \cap C \tag{4.3}$$

or, in terms of membership functions,

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x) \tag{4.4}$$

Note that fuzzy decision D is convex since G and C are convex. The operator "$\wedge$" is minimum operator and the decision is min type fuzzy decision. Min type fuzzy decision represents safety-first approach.

More generally, suppose that we have n goals, $G_1,\ldots,G_n$, and m constraints, $C_1,\ldots,C_m$, then the resultant decision is the intersection of the n goals and m constraints. That is,

$$D = G_1 \cap G_2 \cap \ldots \cap G_n \cap C_1 \cap C_2 \cap \ldots C_m \tag{4.5}$$

and

$$\mu_D = \mu_{G_1} \wedge \ldots \wedge \mu_{G_n} \wedge \mu_{C_1} \wedge \ldots \wedge \mu_{C_m} \tag{4.6}$$

In a broader sense,

Decision = Confluence of goals and constraints

In the confluence diagram in Fig. 4.3, notice that the highest value of $\mu_D(x)$ could be less than 1. This means that there is no option which fully satisfies both the fuzzy goal and fuzzy constraint. It is reasonable in many instances to choose that x or x's which have maximal grade of membership in D. Therefore, the optimal (or maximizing) decision is defined as

$$\mu_D(x^*) = \max_{x \in X} \mu_D(x) \tag{4.7}$$

4.2.2. Deterministic System Under Control

Let us assume that the system under control is deterministic and its temporal evolution is given by the state transition equation

$$x_{t+1} = f(x_t, u_t), \quad t = 0, 1, \ldots \qquad (4.8)$$

where x_t, $x_{t+1} \in X = \{x\}$ is the state at the control stage t and t+1 respectively, and $u_t \in U = \{u\}$ is the control at time t. The initial state is x_0 and the termination time, which is fixed, is $N < \infty$.

4.2.3. Multi-Stage Decision-Making in a Fuzzy Environment

In multi-stage decision-making, there are specified control space $U = \{u\}$ and the state space $X = \{x\}$.

In the beginning, we are in some initial control $u_0 \in U$ which is subjected to a fuzzy constraint $\mu_C^0(u_0)$ and proceed to a fuzzy state $x_1 \in X$ through some known cause effect relationship representing the system under control S; on x_1, a fuzzy goal $\mu_c^1(u_1)$ and attain x_2 on which $\mu_G^2(x_2)$ is imposed and so on. See Fig. 4.4.

Now, to obtain a set of optimum alternatives $(u_0^*, \ldots, u_{N-1}^*)$, we have

$$\mu_D(u_0^*, \ldots, u_{N-1}^*) = \max_{u_0, \ldots, u_{N-1}} (\mu_C^0(u_0) \wedge \ldots \wedge \mu_c^{N-1}(u_{N-1}) \wedge \wedge \mu_G^N(f(x_{N-1}, u_{N-1}))) \qquad (4.9)$$

Notice that the fuzzy goal G^N is imposed on the final state. The last two right-hand-side terms, i.e., $\mu_C^{N-1}(u_{N-1})$

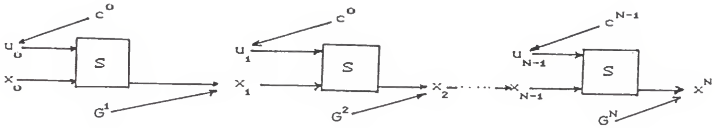


Fig. 4.4. Control-State diagram

and $\mu_G N(f(x_{N-1},u_{N-1}))$ depend only on the control u_{N-1} and not on the other (previous) controls.

The maximization in 4.9 may be split into two parts: over the control sequence $u_0,\dots,u_{N-2}$ and over u_{N-1}, i.e.

$$\mu_D(u_1^*,\dots,u_{N-1}^*|\,x_0) = \max_{u_0,\dots u_{N-2}} [\mu_C^0(u_0) \wedge \dots \wedge \mu_C^{N-2}(u_{N-2}) \wedge \max_{u_{N-1}} (\mu_C^{N-1}(u_{N-1}) \wedge \mu_G^N(f(x_{N-1},\, u_{N-1}))))] \tag{4.10}$$

Since the term $\mu_C^{N-2}(u_{N-2})$ depends only on the control u_{N-2}, then

$$\mu_D(u_0^*,\dots,u_{N-1}^*|x_0) = \max_{u_0,\dots,u_{N-3}} [\mu_C^0(u_0) \wedge \dots \wedge \mu_C^{N-3}(u_{N-3}) \wedge \max_{u_{N-2}} (\mu_C^{N-2}(u_{N-2}) \wedge \max_{u_{N-1}} (\mu_C^{N-1}(u_{N-1}) \wedge \mu_G^N(f(x_{N-1},\, u_{N-1})))] \tag{4.11}$$

On repeating this backward iteration, which is the characteristic of dynamic programming, the following recurrence equations are obtained.

$$\mu_G^{N-i}(x_{N-i}) = \max_{u_{N-i}} (\mu_C^{N-i}(u_{N-i}) \wedge \mu_G^{N-i+1}(x_{N-i+1})) \tag{4.12}$$

and

$$x_{N-i+1} = f(x_{N-i},\, u_{N-i}) \tag{4.13}$$

The optimum sequence of controls to be determined is given by successively maximizing values of u_{N-i} in (4.12).

The above model of dynamic programming recurrence equations were formulated by Bellman and Zadeh in [3] and is often known as Bellman-Zadeh's model of multi-stage decision-making in a fuzzy environment.

4.2.4. A Simpler Approach For Forward Flow

- Branch-And-Bound Method

Like any other dynamic programming models, Bellman and Zadeh's model suffers from the "curse of dimentionality" associated with dynamic programming. However, the model is most suitable for backward flow problems. Here, the total number of options would multiply as the number of alternatives in each stage. Therefore it is not suitable for solving problems with forward flow, i.e. problems that progress with time as it would be cumbersome to handle large dimensions.

Kacprzyk [10] proposed a simpler version which overcomes this difficulty and is known as branch-and-bound method.

The procedure starts from the initial state x_o. A control u_o is applied and the state proceeds to x_1 and so on. Finally, the state x_N is arrived from x_{N-1} under u_{N-1}. For a finite control space this process may be represented by a decision tree (Fig. 4.5). Let us consider a simple example to illustrate this.

Example: If $U = \{c_1, c_2\}$ and $N = 2$, then the decision tree is as shown in Fig. 4.5. The nodes are associated with the particular x_i, $i = 1,2$ obtained via the state transition equation 4.8 while the edges -with values of controls applied at the particular control stages.

To each path in the decision tree, i.e. to each sequence of edges leading from x_o to x_2, representing the consecutive controls applied and states attained, there corresponds some value of

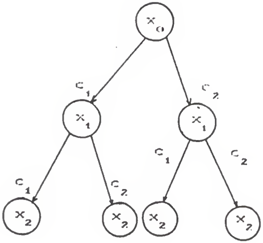


Fig. 4.5. Decision tree in branch-and-bound method.

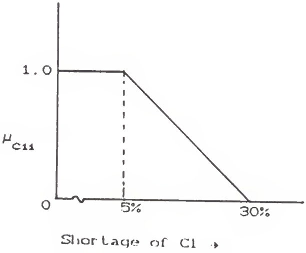


Fig. 4.6. Reference graph for shortage of C1

$$\mu_D(x_0,\ u_1\ x_0) = \mu_C^0(u_0) \wedge \mu_C^1(u_1) \wedge \mu_G^1(x_2) \tag{4.14}$$

The problem is now to find an optimum sequence of controls u_0^*, u_1^*, maximizing (4.14). Due to the finiteness of the control space assumed, the problem reduces to find $u_0^*, \ldots, u_{N-1}^*$ (in the general case) such that

$$\mu_D(u_0^*, \ldots, u_{N-1} | x_0) = \max_{u_0, \ldots, u_{N-1}} (\mu_C^0(u_0) \wedge \ldots \wedge \mu_C^{N-1}(u_{N-1}) \wedge \wedge \mu_G^N(x_N))$$

is equivalent to finding a path in a finite decision tree. The solution based on the full enumeration of all possible paths is possible. But such a procedure is highly inefficient. This "blind" traversal of the decision tree is cumbersome and some implicit enumeration technique is needed which neglects non-promising paths and traverses a possibly a small set of promising ones.

Consider the decision tree as shown in the Fig. 4.5. Here, we traverse the decision tree from x_0 through the consecutive edges (controls) and nodes (states). The essence of the partial enumeration mechanism is basically equivalent to answering the following question: If we currently arrive at some node (state), then to which node (out of those traversed so far) should we most rationally add next edges (controls) to best continue the tree traversal? Or in other words, what is the best continuation of the search process ?

Let us first denote

$$v_0 = \mu_C^0(u_0)$$

....

$$v_k = \mu_C^0(u_0) \wedge \dots \wedge \mu_C^k(u_k) = v_{k-1} \wedge \mu_C^k(u_k) \qquad (4.15)$$

....

$$v_{T-1} = \mu_C^0(u_0) \wedge \dots \wedge \mu_C^{T-1}(u_{T-1}) = v_{T-2} \wedge \mu_C^{T-1}(u_{T-1})$$

$$v_T = \mu_C^0(u_0) \wedge \dots \wedge \mu_C^{T-1}(u_{T-1}) \wedge \mu_G^T(x_T)$$

$$= v_{T-1} \wedge \mu_G^T(x_T) = \mu_D(u_0, \dots, u_{T-1}|x_0) \qquad (4.16)$$

Notice that if we consider some sequence of controls $u_0, \dots u_k < T - 1$, then due to an evident property of "$\wedge$" operator, we have for each $k < w < T - 1$

$$v_k \geqslant v_v = v_k \wedge \mu_C^{k+1}(u_{k+1}) \wedge \dots \wedge \mu_C^v(u_v) \qquad (4.17)$$

because by adding to v_k any further terms through "$\wedge$" we can not increase the value of v_v. We also have

$$v_k \geqslant v_T = \mu_D(u_0, \dots, u_{T-1}|x_0).$$

Thus, let us assume that we are at k^{th} control stage and have traversed so far some nodes. We look now for a node, among those attained so far, to add the edges (to apply controls). Obviously, the node which has the greatest value of v_i should be chosen where $i \in \{1, \dots, k\}$. The other ones do not lead to any optimal solution because for any next controls added, they obviously can not give any better value of v_2.

4.3. Solution for Deterministic System Under Control

This solution approach assumes a deterministic system under control in a fuzzy environment with fuzzy goals and fuzzy constraints. To solve the problem of alternative components already described earlier, we must specify the particular fuzzy constraints and the objective fuzzy goals.

To maintain simplicity, it is desirable to assume the membership function of the fuzzy goals and constraints in piecewise linear form. In this form, their definition requires the specification of two values only: the aspiration level, and the lowest or highest possible value. Needless to say, these values are the result of subjective evaluation made by the experts in that particular area or the decision-maker himself.

Goal: Maximize Z after n stages (membership grade μ_G^N)

Constraints:

1. Shortage (as a percentage of demand) of each alternative component must be as low as possible (μ_{C1i}).

2. Quantity of the main component C2 should be as large as possible (μ_{C2}).

3. Overall machine utilization after n stages for all the l machines should be as high as possible (μ_{C3l}).

Reference graphs for the above goals and constraints could be established by specifying the upper and lower limits (aspiration levels and highest/lowest values). In the graph, the upper limit or aspiration level has the highest grade of membehrship, i.e., 1, and the lower limit has the grade of membership of zero. Grades of memberships of intermediate levels of goals and constraints can be obtained by interpolation using the reference graphs. Assuming a linear relationship, in a reference graph, the upper and lower limits are joined by a straight line. Fig. 4.6 to Fig. 4.12 show the reference graphs for all the goals and constraints.

$$\text{Now, } \mu_D(u_1,\ldots,u_N) = \max_{u_1,\ldots,u_N} (\mu^0_{C11} \wedge \mu^0_{C12} \wedge \mu^0_{C13} \wedge \mu^0_{C2}) \wedge \wedge (\mu^1_{C11} \wedge \mu^1_{C12} \wedge \mu^1_{C13} \wedge \mu^1_{C2}) \wedge \ldots \wedge (\mu^N_{C31} \wedge \mu^N_{C32}) \wedge \mu^N_G \qquad (4.18)$$

Notice that the number of alternatives in each stage depend upon the production plan for each period. Also the constraint of overall machine utilization and the goal are imposed after N stages.

For the sake of simplicity, we shall consider only two stages as the dimension of the problem increases with the number of stages.

Calculations:

Stage 0, $u_0 = 1$:

Constraint 1 (minimize shortages of all alternative components):

Looking at the table 2.4, we get

shortage of C1 = 0 = 0% ⇒ $\mu^0_{C11} = 1$ (from reference graph 4.6)

shortage of C2 = 10 = 1.5% → $\mu^0_{C12} = 1$ (from refernce graph 4.7)

shortage of C3 = 10 = 5% → $\mu^0_{C13} = 1$ (from reference graph 4.8)

Constraint 2 (μ^0_{C2}):

Looking at the table 2.4, we get

quantity of C2 manufactured = 640 = 64%

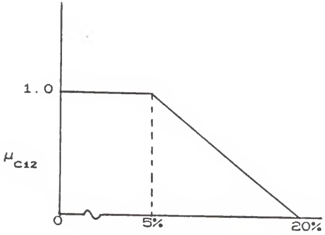
Interpolating using the reference graph 4.9, we get

$\mu^0_{C2} = 0.6$

Similarly, for $u_0 = 2$, we have

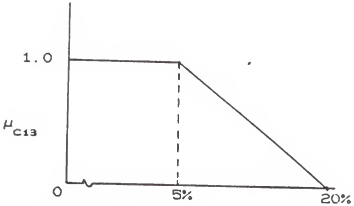
$\mu^0_{C11} = 1$, $\mu^0_{C12} = 1$, $\mu^0_{C13} = 0.5$, $\mu^0_{C2} = 0.63$.

for $u_0 = 3$, we have



Shortage of C2 →

Fig. 4.7. Reference graph jfor for shortage of C2



Shortage of C3 →

Fig. 4.8. Reference graph for shortage of C3

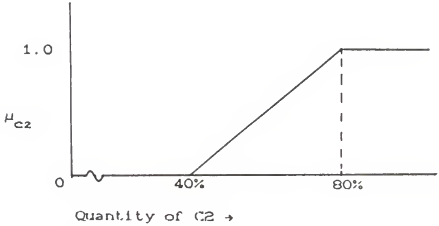


Fig. 4.9. Reference graph for quantity of C2 produced

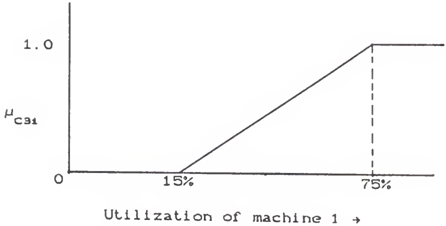


Fig. 4.10. Reference graph for utilization of machine 1

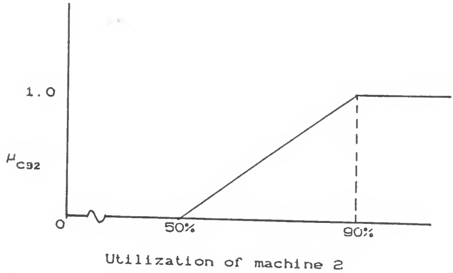


Fig. 4.11. Reference graph for utilization of machine 2

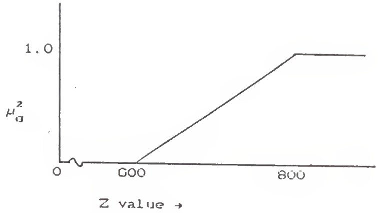


Fig. 4.12. Reference graph for Z value

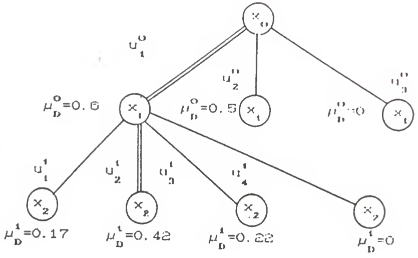


Fig. 4.13. Decision tree for the problem of alternative components

$\mu^{o}_{C11} = 1$, $\mu^{o}_{C12} = 1$, $\mu^{o}_{C13} = 0$, $\mu^{o}_{C2} = 0.65$.

Now, using the equation 4.18, we have

$$\mu^{o}_{D}(u_1, u_2, u_3) = \max_{u_1,u_2,u_3} [\mu^{o}_{C11} \wedge \mu^{o}_{C12} \wedge \mu^{o}_{C13} \wedge \mu^{o}_{C2}]$$

and $\mu^{o}_{D}(u_1) = 1 \wedge 1 \wedge 1 \wedge 0.6 = 0.6$

$\mu^{o}_{D}(u_2) = 1 \wedge 1 \wedge 0.5 \wedge 0.63 = 0.5$

$\mu^{o}_{D}(u_3) = 1 \wedge 1 \wedge 0 \wedge 0.65 = 0$

$\max (\mu^{o}_{D}(u_1,u_2,u_3)) = \max (0.6,0.5,0)$

$= 0.6$

which corresponds to u_1.

According to the branch-and-bound method, the most promising alternative is u_1. So we traverse along this path.

Stage = 1:

Here, we assume that the shortages from the current period are not carried forward to the next period. We assume that the orders not met are lost. Refer to table 4.1.

Assume that $v^{1}_{D} = \mu^{1}_{C11} \wedge \mu^{1}_{C12} \wedge \mu^{1}_{C13} \wedge \mu^{1}_{C2}$

$v^{1}_{D}(u_1) = 1 \wedge 1 \wedge 1 \wedge 0.58 = 0.58$

$v^{1}_{D}(u_2) = 1 \wedge 1 \wedge 0.78 \wedge 0.61 = 0.61$

$v^{1}_{D}(u_3) = 1 \wedge 1 \wedge 0.22 \wedge 0.64$

$v^{1}_{D}(u_4) = 1 \wedge 1 \wedge 0 \wedge 0.67 = 0$

Now, $\mu^{1}_{D}(u_1,u_2,u_3,u_4) = \max (v^{1}_{D} \wedge \mu^{1}_{C31} \wedge \mu^{1}_{C32} \wedge \mu^{2}_{G})$

Overall machine utilization factor (for u_1) =

$$\frac{\text{Plan for period 1 + plan for period 2}}{\text{Machine capacity for 2 periods}}$$

Percentage utilization of machine 1 = $\frac{600 + 600}{1200}$ = 100%

$\mu^{1}_{C31}(u^{o}_{1},u^{1}_{1}) = 1$ (from reference graphs 4.10 and 4.11)

% utilization for machine 2 (for u_1) = $\frac{400 + 300}{1200}$ = 58%

$\mu^1_{C32}(u^0_1,u^1_1) = 0.17.$

Similarly,

$\mu^1_{C31}(u^0_1,u^1_2) = 1, \quad \mu^1_{C32}(u^0_1,u^1_2) = 0.42,$

$\mu^1_{C31}(u^0_1,u^1_3) = 1, \quad \mu^1_{C32}(u^0_1, u^1_3) = 0.63$

$\mu^1_{C31}(u^0_1,u^1_4) = 1, \quad \mu^1_{C32}(u^0_1,u^1_4) = 0.83.$

also, $Z^2 = Z^0 + Z^1$.

$Z^2(u^0_1,u^1_1) = 732 \rightarrow \mu^1_G(u^0_1,u^1_1) = 0.66$ (from the reference graph 4.12).

Similarly,

$\mu^1_G(u^0_1,u^1_2) = 0.74, \quad \mu^1_G(u^0_1,u^1_3) = 0.81, \quad \mu^1_G(u^0_1,u^1_4) = 0.89.$

$\therefore \mu^1_D(u^0_1,u^1_1) = 0.58 \wedge 1 \wedge 0.17 \wedge 0.66 = 0.66$

$\mu^1_D(u^0_1,u^1_2) = 0.61 \wedge 1 \wedge 0.4 \wedge 0.74 = 0.42$

$\mu^1_D(u^0_1,u^1_3) = 0.22 \wedge 1 \wedge 0.63 \wedge 0.81 = 0.22$

$\mu^1_D(u^0_1,u^1_4) = 0 \wedge 1 \wedge 0.83 \wedge 0.89 = 0$

max { 0.17, 0.42, 0.22, 0} = 0.42

$\therefore \{u^*\} = \{u^0_1, u^1_2\}$

Table 4.1. Dynamic Programming Table for Stage 1.

Alternatives (k)	Quantity produced this period			Z	Inventory remaining after consumption			Previous Z	Overall Z
	C1	C2	C3		C1	C2	C3		
1	150	570	180	342	35	-15	0	390	732
2	155	580	165	357	40	-5	-15	390	747
3	160	590	150	372	45	5	-30	390	762
4	165	600	135	387	50	15	-45	390	777

CHAPTER 5

FUZZY SYSTEM UNDER CONTROL IN A FUZZY ENVIRONMENT

5.1. Introduction

In the previous chapter, we saw that in the case of deterministic system under control the initial state, final state and control were non-fuzzy (crisp). Therefore we could determine the grade of membership in the fuzzy goal directly. However, in most practical situations, both the state and control may not be exactly known. In such cases, the fuzziness extends to the control and state variables and the resulting system is called the fuzzy system under control in a fuzzy environment. This chapter discusses the methods available to extend Bellman-Zadeh's model to such systems. Two methods proposed by Baldwin and Pilsworth [1] and Kacprzyk [10] are described. The problem of alternative components is solved using the latter method with some modifications.

5.2. Fuzzy System Under Control

In fuzzy system under control, both the initial state and controls are fuzzy, and hence the final state obtained becomes fuzzy. So, the grade of membership in the fuzzy goal can not be determined in a direct way. Some kind of "trickery" is needed to circumvent this problem in order to obtain the decision in terms if a point value of grade of membership. Both the methods presented here involve some kind of trickery.

5.2.1. Method of Baldwin and Pilsworth

This method extends the Bellman-Zadeh model to include fuzzy state mapping which allows the system dynamics to be described imprecisely.

Some basic definitions:

1. A fuzzy mapping f: $X \to Y$ is a fuzzy set on $X \times Y$ with membership function $\mu_f(x,y)$.

2. A fuzzy function $f(x)$ is a fuzzy set on Y with membership function $\mu_{f(x)}(y) = \mu_f(x,y)$.

3. The fuzzy set $f(A)$ on Y is a fuzzy mapping of a fuzzy set defined as $\mu_{f(A)}(y) = \max_{x \in X} \{\min (\mu_A(x), \mu_f(x,y)), \forall y \in Y\}$.

Consider the finite fuzzy automation. Given:

(i). $J = \{U, X, f, F(U), F(X)\}$ with final state $x(T)$, where U and X are finite spaces for the decision and state sets, respectively, and f: $X \times U \to X$ and $F(U)$, $F(X)$ are the sets for the fuzzy decision and states, respectively.

(ii) a fuzzy dynamical transition equation:

$$\mu_{x(t+1)}(x(t+1)) = \mu_{f(k)} = \max \{\min (\mu_j(k),$$
$$\mu_f(k,x(t+1)))\}, \qquad (5.1)$$

$\forall\, x(t+1) \in X$ and $t = 0,1,\ldots,T-1$ where $J \equiv X \times U$; $k \in K$ and J is a fuzzy set on K representing fuzzy state $x(t)$ and fuzzy decision $u(t)$ with membership function $\mu_j(x,u) = \min (\mu_{x(t)}(x), \mu_{u(t)}(u))$.

Also consider fuzzy constraints $\{\mu_0, \mu_1, \ldots, \mu_{T-1}\}$, $\mu_i \in F(U)$, where μ_i is imposed on the input u_i, $0 \leq i \leq T-1$ and also that a fuzzy goal μ_G^T is imposed on the final state x(T).

Define a relational matrix $R(u,x) = \mu_0(u) \wedge \mu_G^T(x)$; $u \in U$; $x \in X$. This can be thought of as representing how well the control and goal constraints are satisfied. Thus, for a fuzzy control u_0 and resulting state x_1, we can form

$$T(u_0 R\, x_1) = u_0 \circ R \circ x_1 \qquad (5.2)$$

where $\circ$ represents max-min composition as given in 3.2.6. The decision problem is then to choose $u \in U^*$ (the allowed set of fuzzy decisions) such that $T(u_0 R\, x_1)$ is maximized. Thus,

$$T(u_0 R\, x_1) = \max_{x\in X} [\{\max_{u\in U} (\mu_G^T(x) \wedge \mu_0(u) \wedge \mu_{u_0}(u))\}] \wedge \mu_{x_1}(x) \qquad ......(5.3)$$

$$= (\mu_G^T \circ \mu_{x_1}) \wedge (\mu_0 \circ \mu_{u_0}) \qquad (5.4)$$

according to lemma 1 in [1].

Define a functional $\xi_k(x_k) = \max_{u_k\in U_k^*} T(u_k,...u_{T-1} R\, x_T)$, (5.5)

given that the system starts at stage k in fuzzy state x_k with x_T the fuzzy terminal state resulting from a sequential application of fuzzy decisions $u_k,...,u_{T-1}$ under a fuzzy mapping f. By invocation of principle of optimality, a general recurrence equation is obtained as below:

for k = T-1, T-2,..., 1, 0:

$$\xi_k(x_k) = \max_{u_k\in U_k^*} [(\mu_k \circ \mu_{u_k}) \wedge \xi_{k+1}(x_{k+1})]; \qquad (5.6)$$

and for k = T, the terminal stage:

$$\xi_T(x_T) = (\mu_G^T * \mu_{x_T}) \qquad (5.7)$$

Note that for fuzzy subset x_{k+1} of (5.6) the membership function $\mu_{x_{k+1}}(x_{k+1})$ is given by:

$$\mu_{x_{k+1}}(x_{k+1}) = \max_{x_k, u_k \in X \times U} (\mu_{x_k}(x_k) \wedge \mu_{u_k}(u_k) \wedge (\mu_f(x_k, u_k; x_{k+1}))$$

$$= \mu_{x_k}(x_k) \wedge \mu_{u_k}(u_k) \circ (x_f(x_k, u_k; x_{k+1})) \qquad (5.8)$$

It can be seen that dynamic programming functional equation for this problem with fuzzy mapping and initial state in addition to fuzzy environment has associated with it the well known "curse of dimensionality". To overcome this difficulty, Baldwin and Pilsworth propose a process called "fuzzy interpolation" [1]. This is to find a relational matrix for $\xi_{k+1}(x_{k+1})$ using such statements as "if x_{k+1} is large then $\xi_{k+1}(x_{k+1})$ is small", etc. where large and small are fuzzy linguistic variables. With this, a fuzzy value of $\xi_{k+1}(x_{k+1})$ can be determined for any given x_{k+1}. However, the method becomes very cumbersome and involving if there are a large number of state and control variables present in the system. Fuzzy interpolation is dealt with more detail in [1].

5.2.2. Method of Kacprzyk - Branch-And-Bound Method

Kacprzyk proposed a much simpler method for fuzzy system under control using branch-and-bound method in [10]. The method is as follows:

Consider the same fuzzy automata described in the previous section where the fuzzy transition equation is

$$X_{t+1} = F(X_t, U_t)$$

where $X_t, X_{t+1} \subset X = \{x_1, \ldots, x_n\}$ is the fuzzy state at time t and t+1 respectively and $u_t \in U = \{u_1, \ldots, u_n\}$ is a non-fuzzy control, is a particular case of the fuzzy system under control. The temporal evolution of the fuzzy state at the end of stage t is given by

$$\mu_{X_{t+1}}(x_{t+1}) = \max_{x_t} \left(\mu_{X_t}(x_t) \wedge \mu_{X_{t+1}}(x_{t+1}\ x_t, u_t)\right) \qquad (5.9)$$

At each control stage t, the control u_t is subjected to a fuzzy constraint $\mu_G^T(x_T)$ is imposed on the final state $x_t \subset X$. The initial fuzzy state is $x_o \in X$. The branch-and-bound technique works the way it is described in section 4.2.4.

The final fuzzy state x_T can not be introduced directly into $\mu_G^T(x_T)$ as the constraints are of point values and both the final state and goal are fuzzy sets (remember that the control is non-fuzzy). Therefore some trickery is needed to apply the general framework in case of this fuzzy system. It is essential to attain the final fuzzy state x_T as close as possible to the fuzzy goal G^T. A measure of closeness (similarity) between x_T and G^T does in fact serve the same purpose as $\mu_G^T(x_T)$ for a non-fuzzy final state x_T, provided that it takes on a value from [0,1] with 1 indicating that x_T and G^T are the same, zero indicating that they are fully different, and intermediate values indicating varying degrees of differences.

Kacprzyk proposed normalized linear (Hamming) distance which is

$$d_1(x_T, G^T) = 1/n \sum \mu_{x_T}(s_i) - \mu_G^T(s_i) \tag{5.12}$$

where s_i are the states.

For the normalized distance the following relation holds good: if $x_T = G^T$, then $d(x_T, G^T) = 0$, and the more x_T differs from G^T, the closer $d(x_T, G^T)$ to 1. If $\mu_{G2}^T(x_T)$ is the measure of closeness of final fuzzy state to fuzzy goal, then

$$\mu_{G2}^T(x_T) = 1 - d(x_T, G^T) \tag{5.13}$$

Then, the fuzzy decision becomes

$$\mu_D(u_0^*, \ldots u_{T-1}^* | x_0) = \max_{u_0, \ldots, u_{T-1}} (\mu_c^0(u_0) \wedge \ldots \wedge \mu_c^{T-1}(u_{T-1}) \wedge \\ \wedge \mu_{G2}^T(x_T) \tag{5.14}$$

The case of fuzzy control may be dealt with by the approximation of the fuzzy controls applied by the reference fuzzy controls, so as a finite number of possible controls exist which is a pre-requisite to construct and handle the decision tree.

5.3. Difficulties in Both the Approaches

Both approaches, Baldwin and Pilsworth's and Kacprzyk's, suffer from a difficulty. The basis of both the methods is the fuzzy state mapping. To obtain this mapping, one has to know all possible states the system goes through [23]. This is hardly the case in actual practice. In most cases, the decision maker is not aware of all possible states. In most of the backward flow problems, for example, resource allocation, travelling salesman, and network problems, it is possible to enumerate all possible states. But it is extermely difficult and often impossible to enumerate all possible states in most of the forward flow (progressive in time) problems with the exception of a few simple cases of inventory problems. Also, solving the problem by both the approaches becomes tedious if the number of all possible states is extremely large. In such cases, one has to look for a more intuitive and simpler approaches in order to solve the problem of fuzzy system under control using Bellman-Zadeh model. The problem which we are currently solving falls into this category.

5.4. Solution to the Problem - Fuzzy System Under Control

In this approach, for the sake of simplicity, we assume non-fuzzy control. In order to eliminate unnecessary computations, we use the branch-and-bound method with some modifications as the original method described in the previous section can not be applied directly to solve our problem. The major features of our problem in the framework of fuzzy system under control are:

1. The percentage production factors of alternative components are fuzzy sets. This results in fuzzy initial state.
2. All constraints except that of machine utilization (overall) are imposed on state variables. The constraint of overall machine utilization is applied on the control.

3. The reference sets for overall goal and constraints are same as in the section 4.2.3.

4. As we shall see later, the state variables, after imposing constraints, yield type 2 fuzzy sets - which are difficult to handle

5. Since we have assumed non-fuzzy control, the grades of membership of constraints imposed on control (overall machine utilization) have point values whereas the state variables are fuzzy and do not have point values of their grades of membership.

Given:

Two machines with production capacity of 600 per period each and a minimum batch quantity of 400 each.

Production plan:

Period	Quantity
1	1000
2	900

Demand: C1 - 15%, C2 - 65%, C3 - 20%, non-fuzzy.

Period	Quantity
1	1000
2	900

Production alternatives for the period 1

	u_1	u_2	u_3
Machine 1	400	500	600
Machine 2	600	500	400

Percentage production factors for alternative components:

Machine 1:

$s_{i1} =$

Component μ	1	0.7	0.3
C1	15	20	15
C2	55	60	60
C3	30	20	25

Machine 2:

$s_{i2} =$

Component μ	1	0.7	0.3
C1	10	15	20
C2	70	60	65
C3	20	25	15

Note that for the sake of simplicity we have chosen same membership function values for all alternative components. This would make the computations easier.

Calculations:

Stage 0, $u_1 = 1$:

Total quantity to be manufactured on machine 1 = 400

Total quantity to be manufactured on machine 2 = 600

Now, the quantity of C1, C2, and C3 manufactured on machine 1

$= Q_i \times s_{i1} = q_{ijk}$

$= 400 \times s_{i1} =$

comp μ	1	0.7	0.3
C1	60	80	60
C2	220	240	240
C3	120	80	100

Similarly, for machine 2, we have

$q_{i11} = 600 \times s_{i1} =$

comp μ	1	0.7	0.3
C1	60	90	120
C2	420	360	390
C3	120	150	90

Total quantity of each alternative components made = Quantity made on machine 1 + quantity made on machine 2

Comp μ	1	0.7	0.3
C1	120	170	180
C2	640	600	630
C3	240	230	190

which is equivalent in fuzzy notation,

$q_{111} = 1/120 + 0.7/170 + 0.3/180$

$q_{211} = 1/640 + 0.7/600 + 0.3/630$

$q_{311} = 1/240 + 0.7/230 + 0.3/190$

Similarly, the total quantity of each alternative component made for u_2 and u_3 are:

For u_1:

$q_{112} = 1/125 + 0.7/175 + 0.3/175$

$q_{212} = 1/625 + 0.7/600 + 0.3/625$

$q_{312} = 1/250 + 0.7/225 + 0.3/200$

For u_3:

$q_{113} = 1/130 + 0.7/180 + 0.3/170$

$q_{213} = 1/610 + 0.7/600 + 0.3/620$

$q_{313} = 1/260 + 0.7/220 + 0.3/210$

Now, subtracting the demand (C1: 150, C2: 650, C3: 200) from the total quantities, we get inventory at the end of period 1, i.e., $I_{i1k} = \sum_{l=1}^{2} q_{ijk} - d_{ij}$

for u_1:

C1: 1/-30 + 0.7/25 + 0.3/30

I_{i11} = C2: 1/-10 + 0.7/-50 + 0.3/-20

C3: 1/40 + 0.7/30 + 0.3/-10

for u_2:

C1: 1/-25 + 0.7/25 + 0.3/25

I_{i12} = C2: 1/-25 + 0.7/-50 + 0.3/30

C3: 1/50 + 0.7/25 + 0.3/0

for u_3:

C1: 1/-20 + 0.7/30 + 0.3/20

I_{i13} = C2: 1/-40 + 0.7/-50 + 0.3/30

C3: 1/60 + 0.7/20 + 0.3/10

Now, using the reference graphs 4.6, 4.7, 4.8 for constraint 1 (minimize shortages), we get

for u_1:

$\mu^{o}_{C11}(u^{o}_{1})$ = 1/0.4 + 0.7/1 + 0.3/1

$\mu^{o}_{C12}(u^{o}_{1})$ = 1/1 + 0.7/0.82 + 0.3/1

$\mu^{o}_{C13}(u^{o}_{1})$ = 1/1 + 0.7/0.82 + 0.3/1

for u_2:

$\mu^{o}_{C11}(u^{o}_{2})$ = 1/0.67 + 0.7/1 + 0.3/1

$\mu^{o}_{C12}(u^{o}_{2})$ = 1/1 + 0.7/0.82 + 0.3/1

$\mu^{o}_{C13}(u^{o}_{2})$ = 1/1 + 0.7/1 + 0.3/1 for u_3:

$\mu^o_{C11}(u^o_3) = 1/0.67 + 0.7/1 + 0.3/1$

$\mu^o_{C12}(u^o_3) = 1/0.92 + 0.7/0.82 + 0.3/1$

$\mu^o_{C13}(u^o_3) = 1/1 + 0.7/1 + 0.3/1$

Note that the above fuzzy sets are of type 2

Similarly, using the reference graph 4.9 for constraint 2 (maximize the quantity of C2), we get

$\mu^o_{C2}(u^o_1) = 1/0.6 + 0.7/0.5 + 0.3/0.58$

$\mu^o_{C2}(u^o_2) = 1/0.56 + 0.7/0.5 + 0.3/0.58$

$\mu^o_{C2}(u^o_3) = 1/0.53 + 0.7/0.5 + 0.3/0.55$

Now, for the first stage,

$$\mu^o_D(u^o) = \max_{u_1,u_2,u_3} (\mu^o_{C11} \wedge \mu^o_{C12} \wedge \mu^o_{C13} \wedge \mu^o_{C2})$$

$$\mu^o_D(u^o_1) = (1/0.4 + 0.7/1 + 0.3/1) \wedge (1/1 + 0.7/0.82 + 0.3/1) \wedge$$
$$\wedge (1/1 + 0.7/0.82 + 0.3/1) \wedge (1/0.6 + 0.7/0.5 + 0.3/0.58).$$

Here, we use the extension principle for min operator on type 2 fuzzy sets (already described in 3.2.7.). A sample calculation of such operation is given below:

$(1/0.4 + 0.7/1 + 0.3/1) \wedge (1/1 + 0.7/0.82 + 0.3/1) =$

$(1 \wedge 1)/(0.4 \wedge 1) = 1/0.4$

$(1 \wedge 0.7)/(0.4 \wedge 0.82) = 0.7/0.4$

$(1 \wedge 0.3)/(0.4 \wedge 1) = 0.3/0.4$

$(0.7 \wedge 1)/(1 \wedge 1) = 0.7/1$

$(0.7 \wedge 0.7)/(1 \wedge 0.82) = 0.7/0.82$

$(0.7 \wedge 0.3)/(1 \wedge 1) = 0.3/1$

$(0.3 \wedge 1)/(1 \wedge 1) = 0.3/1$

$(0.3 \wedge 0.7)/(1 \wedge 0.82) = 0.3/0.82$

$(0.3 \wedge 0.3)/(1 \wedge 1) = 0.3/1$

Now, for 1, we have max $\{0.4\} = 0.4 \Rightarrow 1/0.4$

for 0.7 max $\{0.4, 1, 0.82\} = 1 \Rightarrow 0.7/1$

for 0.3, max $\{0.4, 1, 1, 0.82, 1\} = 1 \Rightarrow 0.3/1$

$$\therefore (1/0.4 + 0.7/1 + 0.3/1) \wedge (1/1 + 0.7/0.82 + 0.3/1) = (1/0.4 + 0.7/1 + 0.3/1)$$

Continuing the same process, we get

$\mu_D^o(u_1^o) = (1/0.4 + 0.7/0.6 + 0.3/0.6)$.

Similarly,

$\mu_D^o(u_2^o) = (1/0.56 + 0.7/0.56 + 0.3/0.56)$

$\mu_D^o(u_3^o) = (1/0.53 + 0.7/0.53 + 0.3/0.55)$

See fig. 5.1.

Now, we have to choose the maximum value of μ_D^o among three alternatives in order to branch off to the next stage. Since all the values of μ_D^o are type 2 fuzzy sets, it is not possible to determine which one has the greatest value. In order to overcome this difficulty, we propose a simple and intuitive method to evaluate the greatest value of three type 2 fuzzy sets.

In order to compare the type 2 fuzzy sets, we need to convert them into a point value. This could act as an index based on which we can compare the type 2 fuzzy sets. This index can be obtained by taking a weighted average of the two grades of memberships. Obviously, the weighted average lies in the interval [0,1]. A value of 1 indicates total crispness and the other values within the interval indicate varying degrees of crispness. Therefore, we can choose that set as the best which has the highest value of crispness index.

Consider a fuzzy set of type 2 as given below.

$A = \{(y_1,\mu(y_1)),\ (y_2,\mu(y_2)),\dots,\ (y_n,\mu(y_n))\}$

where $y_1,y_2,\dots,y_n,\ \mu(y_1),\dots,\mu(y_n) \in [0,\ 1]$, and

$y \in Y,\ y = f(x),\ x \in X.$

Then, crispness index $= \gamma = 1/n \sum_{i=1}^{n} y_i\mu(y_i)$ (5.15)

Now, using the above definition, we get

$\gamma(u_1^o) = 1/3\ (1 \times 0.4 + 0.7 \times 0.6 + 0.3 \times 0.6) = 0.3333$

Similarly,

$\gamma(u_2^o) = 0.3733$ and

$\gamma(u_3^o) = 0.3553$

Since $\gamma(u_2^o) = 0.3733$ is the greatest of the three values, we choose u_2^o. i.e. $u^{o*} = \{u_2^o\}$.

Stage 2.

Production plan for the period 2 is 900. Using the equations 2.8 and 2.9 , we obtain the following production alternatives.

Machine	Alternatives u_1	 u_2	 u_3	 u_4
1	300	400	500	600
2	600	500	400	300

Demand for the period 2 is 900 units, i.e.

d_{12}: 135, d_{22}: 585, d_{32}: 180

Using the percentage production factors for alternative components, we get the following total quantities of each alternative components produced by two machines.

for u_1

$q_{121} = 1/105 + 0.7/150 + 0.3/165$

$q_{221} = 1/585 + 0.7/540 + 0.3/570$

$q_{321} = 1/210 + 0.7/540 + 0.3/570$

for u_2

$q_{122} = 1/110 + 0.7/155 + 0.3/160$

$q_{222} = 1/570 + 0.7/540 + 0.3/565$

$q_{322} = 1/220 + 0.7/205 + 0.3/175$

for u_3

$q_{123} = 1/115 + 0.7/160 + 0.3/155$

$q_{223} = 1/555 + 0.7/540 + 0.3/560$

$q_{323} = 1/230 + 0.7/200 + 0.3/185$

for u_4

$q_{124} = 1/120 + 0.7/165 + 0.3/150$

$q_{224} = 1/540 + 0.7/540 + 0.3/555$

$q_{324} = 1/240 + 0.7/195 + 0.3/195$

Now, assuming that there is no backlogue from the period 1 (i.e. lost sales), and using the equation 2. i.e.

$I_{i2k} = q_{i2k} + I_{i1k} - d_{i2}$ where if $I_{i1k} < 0$, $I_{i1k} = 0$,

we get

for u_1

$I_{121}(u_1^1) = 1/-30 + 0.7/40 + 0.3/55$

$I_{221}(u_1^1) = 1/0 + 0.7/-45 + 0.3/10$

$I_{321}(u_1^1) = 1/80 + 0.7/55 + 0.3/-15$

$I_{122}(u_2^1) = 1/-25 + 0.7/45 + 0.3/50$

$I_{222}(u_2^1) = 1/-15 + 0.7/-45 + 0.3/5$

$I_{322}(u_2^1) = 1/90 + 0.7/50 + 0.3/-5$

$I_{123}(u_3^1) = 1/-20 + 0.7/50 + 0.3/45$

$I_{223}(u_3^1) = 1/-30 + 0.7/-45 + 0.3/0$

$I_{323}(u_3^1) = 1/100 + 0.7/45 + 0.3/5$

$I_{124}(u_4^1) = 1/-15 + 0.7/55 + 0.3/40$

$I_{224}(u_4^1) = 1/-45 + 0.7/-45 + 0.3/-5$

$I_{324}(u_4^1) = 1/110 + 0.7/40 + 0.3/15$

Using the reference graphs for constraints, we get

$\mu_{C11}^1(u_1^1) = 1/0.32 + 0.7/1 + 0.3/1$

$\mu_{C12}^1(u_1^1) = 1/1 + 0.7/0.83 + 0.3/1$

$\mu_{C13}^1(u_1^1) = 1/1 + 0.7/1 + 0.3/0.78$

$\mu_{C11}^1(u_2^1) = 1/0.46 + 0.7/1 + 0.3/1$

$\mu_{C12}^1(u_2^1) = 1/1 + 0.7/0.89 + 0.3/1$

$\mu_{C13}^1(u_2^1) = 1/1 + 0.7/1 + 0.3/1$

$\mu_{C11}^1(u_3^1) = 1/0.61 + 0.7/1 + 0.3/1$

$\mu_{C12}^1(u_3^1) = 1/0.99 + 0.7/0.83 + 0.3/1$

$\mu_{C13}^1(u_3^1) = 1/1 + 0.7/0.83 + 0.3/1$

$\mu_{C11}^1(u_4^1) = 1/0.76 + 0.7/1 + 0.3/1$

$\mu_{C12}^1(u_4^1) = 1/0.83 + 0.7/0.83 + 0.3/1$

$\mu^{1}_{C13}(u^{1}_{4}) = 1/1 + 0.7/1 + 0.3/1$

Constraint 2 (maximize quantity of C2) becomes

$\mu^{1}_{C2}(u^{1}_{1}) = 1/0.63 + 0.7/0.5 + 0.3/0.58$

$\mu^{1}_{C2}(u^{1}_{2}) = 1/0.58 + 0.7/0.5 + 0.3/0.57$

$\mu^{1}_{C2}(u^{1}_{3}) = 1/0.54 + 0.7/0.5 + 0.3/0.56$

$\mu^{1}_{C2}(u^{1}_{4}) = 1/0.5 + 0.7/0.5 + 0.3/0.54$

Overall machine utilization constraint becomes

$\mu^{1}_{C31}(u^{1}_{1}) = 0.86; \ \mu^{1}_{C32}(u^{1}_{1}) = 1$

$\mu^{1}_{C31}(u^{1}_{2}) = 1; \ \mu^{1}_{C32}(u^{1}_{2}) = 0.89$

$\mu^{1}_{C31}(u^{1}_{3}) = 1; \ \mu^{1}_{C32}(u^{1}_{3}) = 0.75$

$\mu^{1}_{C31}(u^{1}_{4}) = 1; \ \mu^{1}_{C32}(u^{1}_{4}) = 0.61$

Note that the grades of membership of overall machine utilization are point values since we have assumed non-fuzzy demand and non-fuzzy control.

Goal:

$Z(u^{0}_{2}) = 1/375 + 0.7/325 + 0.3/365$ for stage 0.

for stage 1, we have

$Z(u^{1}_{1}) = 1/375 + 0.7/294 + 0.7/339$

$Z(u^{1}_{2}) = 1/350 + 0.7/293 + 0.3/332$

$Z(u^{1}_{2}) = 1/325 + 0.7/292 + 0.3/325$

$Z(u^{1}_{4}) = 1/329 + 0.7/291 + 0.3/318$

Overall Z values are

$Z(u^{0}_{2},u^{1}_{1}) = 1/750 + 0.7/619 + 0.3/704$

$Z(u^{0}_{2},u^{1}_{2}) = 1/725 + 0.7/618 + 0.3/697$

$Z(u^{0}_{2},u^{1}_{3}) = 1/700 + 0.7/617 + 0.3/690$

$Z(u^{0}_{2},u^{1}_{4}) = 1/704 + 0.7/616 + 0.3/683$

After using the reference graph for Z value, we have

$\mu_G^1(u_2^0,u_1^1) = 1/0.75 + 0.7/0.1 + 0.3/0.52$

$\mu_G^1(u_2^0,u_2^1) = 1/0.63 + 0.7/0.09 + 0.3/0.49$

$\mu_G^1(u_2^0,u_3^1) = 1/0.5 + 0.7/0.09 + 0.3/0.45$

$\mu_G^1(u_2^0,u_4^1) = 1/0.52 + 0.7/0.08 + 0.3/0.42$

Now,

$$\mu_D^1(u^0,u^1) = \mu_D^0(u_2^0) \wedge \max_{u_1^1,u_2^1,u_3^1,u_4^1} (\mu_{C11}^1 \wedge \mu_{C12}^1 \wedge \mu_{C13}^1 \wedge \mu_{C2}^1 \wedge \mu_{C31}^1 \wedge \mu_{C32}^1 \wedge \mu_G^1)$$

$\mu_D^1(u_2^0,u_1^1) = (1/0.32 + 0.7/1 + 0.3/1) \wedge (1/1 + 0.7/0.83 +0.3/1)$

$\wedge\ (1/1 + 0.7/1 + 0.3/0.78) \wedge (1/0.63 + 0.7/0.5 + 0.3/0.58) \wedge$

$\wedge\ 0.86 \wedge 1 \wedge (1/0.75 + 0.7/0.1 + 0.3/0.52)$

Since the type 2 fuzzy sets follow the associative law, we can put the point values in the end, i.e.

$\mu_D^1(u_2^0,u_1^1) = (1/0.32 + 0.7/1 + 0.3/1) \wedge (1/1 + 0.7/0.83 +0.3/1)$

$\wedge\ (1/1 + 0.7/1 + 0.3/0.78) \wedge (1/0.63 + 0.7/0.5 + 0.3/0.58) \wedge$

$\wedge\ (1/0.75 + 0.7/0.1 + 0.3/0.52) \wedge 0.86 \wedge 1$

$\mu_D^1(u_2^0,u_1^1) = (1/0.32 + 0.7/0.63 + 0.3/0.63) \wedge 0.86 \wedge 1$

Similarly,

$\mu_D^1(u_2^0,u_2^1) = (1/0.46 + 0.7/0.58 + 0.3/0.58) \wedge 1 \wedge 0.89$

$\mu_D^1(u_2^0,u_3^1) = (1/0.5 + 0.7/0.5 + 0.3/0.5) \wedge 1 \wedge 0.75$

$\mu_D^1(u_2^0,u_4^1) = (1/0.5 + 0.7/0.5 + 0.3/0.52) \wedge 1 \wedge 0.61$

Now we calculate the crispness index for the type 2 fuzzy sets to get a point value, i.e.

$\mu_D^1(u_2^0,u_1^1) = 0.3167 \wedge 0.86 \wedge 1 = 0.3167$

$\mu_D^1(u_2^0,u_2^1) = 0.3467 \wedge 1 \wedge 0.89 = 0.3467$

$\mu_D^1(u_2^0,u_3^1) = 0.3333 \wedge 1 \wedge 0.75 = 0.3333$

$\mu_D^1(u_2^0,u_4^1) = 0.3353 \wedge 1 \wedge 0.61 = 0.3353$

Since $\mu_D^1(u_2^0,u_2^1)$ has the highest value with 0.3467, we choose u_2 for period 2. Therefore the final solution is

Stage 0	Stage 1
u_2	u_2
Machine 1: 500	Machine 1: 400
Machine 2: 500	Machine 2: 500

See Fig. 5.2.

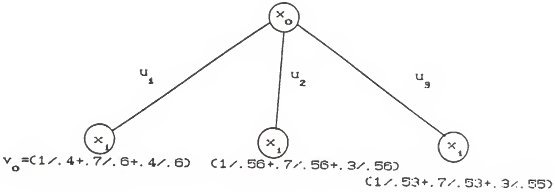


Fig. 5.1. Decision tree for stage 0.

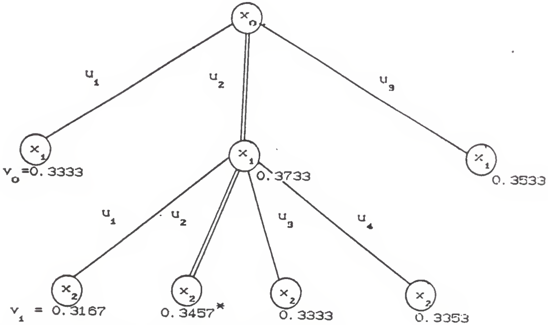


Fig. 5.2 Decision tree for the example.

CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1. Summary

Alternative components are very common in manufacturing environment. They components arise out of lack of total control over manufacturing processes and have same basic dimensions and tolerance but different upper and lower limits. Since alternative components form many options for a single part, it is difficult to plan and control the production of such components. Therefore it is desirable to maximize the production of the main component which is most desirable, whenever alternatives exist, by choosing a proper one. In situations where a part is manufactured on many similar machines each having its own degree of control, it is possible to have different alternative plans by splitting the order quantity among the machines in such a way as to maximize the quantity of the main component produced. Since this decision is an ongoing process over a length of a finite number of periods, it becomes a multi-stage decision-making problem where dynamic programming is extensively used.

This study develops a mathematical model of the problem of alternative components using the dynamic programming approach. The problem is treated under three categories namely:

-deterministic system under control in a crisp environment.

-deterministic system under control in a fuzzy environment.

-fuzzy system under control in a fuzzy environment.

Mathematical models are formulated for each category and an example of the problem is solved in each case. Bellman and Zadeh's model of decision-making in a fuzzy environment is used as a basic framework to study the problem. In case of fuzzy system under control in fuzzy environment, the existing approaches are evaluated in view of the applicability to our problem. It was found that the existing two methods, Baldwin and Pilsworth's method and branch-and-bound method of Kacprzyk, can not be applied to our problem in fullness. The latter method is used to solve our problem with some modifications.

6.2. Discussions

In deterministic system, it is very essential that all the parameters be crisp. As we have seen in chapter 2, conflicting goals and constraints result in a deadlock situation where the decision maker is left with no alternatives to choose from. In most of the practical situations, physical quantities, goals and constraints can not be specified detrministically. In many situations there is vagueness associated with many parameters and variables and the probabilistic models can not handle fuzziness as they are designed to take care of randomness.

Defining the problem in a fuzzy environment alleviates some of the difficulties by allowing the goals and constraints to be defined with imprecision. Imprecise terms like "as low as possible" and "as high as possible" can now be accomodated in the decision model by fuzzifying such parameters. Obviously, the fuzzification is subjective and done by a person who is an expert in that area or the one who has a good knowledge of the syatem. Fuzziness of goals and constraints can be represented by the reference graphs which have an upper limit or an aspiration level and a lower limit or the worst case which are linked by a linear relation. The decision yields a point value of the grade of membership and the alternative which has the highest grade among all the alternatives is chosen as a decision.

Since Bellman and Zadeh's model of multi-stage decision-making in a fuzzy environment is designed for backward flow problems, it can not handle forward flow problems with high dimensions. Kacprzyk's branch-and-bound method offers a simple approach for forward flow problems with high dimensionality. Instead of doing a blind traversal of all possible alternatives, it chooses the most promising paths, thus eliminating the unpromising alternatives. Since our problem can have a high dimentionality, if there are more than two machines and the lot sizes are smaller instead of multiples of 100, branch-and-bound method appears to be the most efficient method.

We see in case of fuzzy system under control that, as the number of fuzzy parameters increase, the problem gets more and more complicated. Both the methods presented require a fuzzy state mapping to start with. This means that the decision maker needs to enumerate all possible states to obtain a transition matrix for both state and control. The main problem here is that, in most practical situations it is almost impossible to enumerate all possible states. For example, if the inventory level is considered as a state variable, then the number of all possible levels attained by inventory will be infinitely large. In a very highly uncertain situation, the inventory level may range from 10 units to 100,000 units. In such a case, there would be 99,990 possible states. Suppose if the control variables are 10, then the mapping matrix would be of 999,900 x 999,900 ! Since this is the case in most of the forward flow problems, it would be impossible to apply both the methods to the problem. In order to solve such problems, it is essential to look at the basic underlying philosophy governing the method rather than applying the method in its rigid framework. One has to look for a simple and often heuristic approach to obtain a rational solution for a problem. The crispness index developed in chapter 5 is one of such attempts. We have used the basic concept of the decision being the confluence of goals and constraints in our approach. Since the branch-and-bound method cuts down the number of traversals drastically, this method has been adopted. We can see that even with the assumption of non-fuzzy control we ended up with type 2 fuzzy sets for which methods of comparison are not available. One can imagine the complexity that would have arisen had we used fuzzy control and fuzzy demand. We can say say that our method of obtaining the crispness index by weighting the membership grades of the type 2 fuzzy set is intuitive, appropriate and serves the purpose.

Until now, only two methods are available for fuzzy system under control. Both are applicable for certain specific types of problems where it is possible enumerate all possible states to form a state transitional matrix. However, this study seems to be the first attempt to solve the problem of alternative components. And the limitations of the existing approaches are evidently clear in this study. Much work needs to be done in the direction of obtaining simpler and intuitive methods which can be applied to a wide variety of problems.

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CONTROL OF ALTERNATIVE COMPONENTS IN A FUZZY ENVIRONMENT

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ABSTRACT

Alternative components are very common in manufacturing environment. They are the result of lack of control over manufacturing processes. They have the same basic dimension and tolerance but different upper and lower limits. Since alternative components form many options for a single part, it is difficult to plan and control the production of such components and their mating parts. In most of the cases, achieving complete control over processes is impossible or costs are prohibitive and the components are made on many similar machines each having its own degree of control. Then, it is desirable to maximize the production of the main component by suitably allocating the order quantities to the machines. Since this decision is an ongoing process over a length of finite number of periods, it becomes a multi-stage decision-making problem where dynamic programming is extensively used.

This study develops a mathematical model for the problem of alternative components using the dynamic programming approach. The problem is treated under three categories namely:

-deterministic system under control in a crisp

environment.

-deterministic system under control in a fuzzy environment.

-fuzzy system under control in a fuzzy environment.

Mathematical models are formulated for each category and an example of the problem is solved in each case. Bellman and Zadeh model of decision-making in a fuzzy environment is used as a basic framework to study the problem. In case of fuzzy system under control in a fuzzy environment, the existing approaches of Baldwin and Pilsworth and Kacprzyk are evaluated in view of applicability to our problem. They were found not to be completely applicable to our problem in their fullness as the problem falls outside the domain assumed by both the methods. The branch-and-bound method of Kacprzyk is used in solving our problem with a modification in the form of crispness index to evaluate the type 2 fuzzy sets resulting from the fuzzy state.